

Physics 226: Problem Set #5

Due in Class on Thursday October 6

1. **Deep Inelastic Scattering Kinematics** Consider the process $ep \rightarrow e + X$. We are going to derive the expression quoted in class that gives the cross section in terms of the structure functions and the definitions of x and y :

$$\begin{aligned} x &\equiv -\frac{q^2}{2p \cdot q} \\ y &\equiv \frac{p \cdot q}{p \cdot k} \end{aligned}$$

where p is the initial four-momentum of the proton, k is the initial four-momentum of the electron, k' is the final four-momentum of the electron and $q \equiv k - k'$.

- (a) From the definition of y , show that

$$1 - y = \frac{p \cdot k'}{p \cdot k} = \frac{1}{2} (1 + \cos \theta^*)$$

where θ^* is the scattering angle in the e -parton center-of-mass frame.

- (b) Starting with the expression for e -parton elastic scattering in the center-of-mass frame:

$$\frac{d\sigma^{eq}}{d\Omega} = e_q^2 \frac{\alpha^2}{8p^2 \sin^4(\theta/2)} [1 + \cos^4(\theta/2)]$$

where e_q is the charge of the quark or antiquark in units of e (eg $e_q = 2/3$ for up quarks) and p is the incoming momentum of the e in the center-of-mass frame, perform a change of variables to find the differential cross section $d\sigma^{eq}/dy$.

- (c) Use the fact that the deep inelastic ep cross section can be calculated as an incoherent sum over e -parton scattering cross sections to turn your expression from part (b) into an expression for the ep scattering cross section $d^2\sigma^{ep}/dx dy$ in terms of a sum over $f_i(x)$, where $f_i(x)$ is the parton distribution function for parton species i .

(d) Use the definition of $F_2(x)$:

$$F_2(x) \equiv \sum_i e_i^2 x f_i(x)$$

to rewrite $d^2\sigma^{ep}/dxdy$ in terms of $F_2(x)$ instead of the sum over the $f_i(x)$.

2. **Parton Model Kinematics** At large momentum transfer, hadron-hadron scattering can be described using the parton model. An old, but very complete, discussion of this process can be found in Reviews of Modern Physics 56: 579707 (SuperCollider physics). We will use the notation of that article here. The cross section for the reaction $a + c \rightarrow c + X$ is given by

$$d\sigma(a + b \rightarrow c + X) = \sum_{ij} f_i^{(a)}(x_a) f_j^{(b)}(x_b) d\hat{\sigma}(i + j \rightarrow c + X)$$

where $f_i(x)$ is the parton distribution function for partons of species i to carry a fraction x of the proton's momentum (this is the same $f_i(x)$ as in the previous problem) and $\hat{\sigma}(i + j \rightarrow c + X)$ is the hard scattering cross section. Prove the following:

- (a) The invariant mass squared of the hard scattering system $\hat{s} = \tau s$ where s is the center-of-mass energy squared of the hadron-hadron collision and $\tau = x_a x_b$
- (b) The longitudinal momentum of the hard scattering system (ie the momentum along the beamline) is $p = x\sqrt{s}/2$ where $x = x_a - x_b$. Our convention is that particle a comes from the left and particle b from the right with $p_{||a} = x_a\sqrt{s}/2$ and $p_{||b} = x_b\sqrt{s}/2$.
- (c) The kinematic variables x_a and x_b are related to the variables of the hadronic process by

$$x_{a,b} = \frac{1}{2} \left[\left(x^2 + 4\tau \right)^{\frac{1}{2}} \pm x \right]$$